

Regularity of roots of polynomials

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Abstract. We show that smooth curves of monic complex polynomials $P_a(Z) = Z^n + \sum_{j=1}^n a_j Z^{n-j}$, $a_j : I \rightarrow \mathbb{C}$ with $I \subset \mathbb{R}$ a compact interval, have absolutely continuous roots in a uniform way. More precisely, there exist a positive integer k and a rational number $p > 1$, both depending only on the degree n , such that if $a_j \in C^k$ then any continuous choice of roots of P_a is absolutely continuous with derivatives in L^q for all $1 \leq q < p$, in a uniform way with respect to $\max_j \|a_j\|_{C^k}$. The uniformity allows us to deduce also a multiparameter version of this result. The proof is based on formulas for the roots of the universal polynomial P_a in terms of its coefficients a_j which we derive using resolution of singularities. For cubic polynomials we compute the formulas as well as bounds for k and p explicitly.

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